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A SPATIAL AUTOCORRELATION MODEL OF THE EFFECTS OF POPULATION DENSITY ON FERTILITY*

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The relationship between fertility and population density that has been demonstrated with aggregate data is discussed and reexamined here. We argue that models which exclude considerations of spatial processes are incomplete and that therefore parameter estimates for such models are misleading. We then develop a model which incorporates spatial autocorrelation, and reanalyze data from one well-known study of the effects of density on fertility using our model. The results show that, with one exception, the effects of density on fertility are trivial, a finding that is quite different from previous research. We conclude that spatial mechanisms are an important consideration in the modeling of social processes that involve geographically defined units.

INTRODUCTION

The relationship between population density and social pathology has received extensive attention since the early 1970s. Building on ethological work (e.g., Calhoun, 1962), sociologists have developed and tested models using a variety of sampling strategies and methodological techniques. Many of these studies are reviewed and critiqued in an excellent overview of the field provided by Choldin (1978). He is quite critical of the density-pathology literature and concludes skeptically that "the density-pathology hypothesis fails to be confirmed within urban areas" (p. 109).

In effect, the existence of a causal relationship between population density and social pathology seems doubtful on theoretical grounds, but the fact that studies consistently report statistically significant relationships, which critics have not been able to demonstrate to be spurious, has established a prima facie case appealing enough that the findings continue to be widely cited in textbooks as well as in the research literature.[1] The puzzle is why the findings persist when the credibility of the causal relationship is so low. In this paper we propose an answer to the puzzle. We argue that previous estimates are misleading because they are based on models which do not allow for the presence of spatial interaction (autocorrelation) among the units of analysis. We briefly discuss the density-pathology issue, with particular reference to one influential study in the field; then we discuss the link between this issue and spatial autocorrelation. Finally, we analyze data on density and fertility by developing a model which allows for the effects of spatial autocorrelation among disturbances. Our results lead to conclusions which are quite different from those of previous research. Though our analysis is specific to the density-fertility relationship, we do not intend to fuel the debate over this particular substantive issue. Indeed, this issue is probably best investigated with other research designs. Rather, our purpose is to illustrate the effects of spatial autocorrelation on parameter estimates and to suggest the relevance of spatial autocorrelation models for a number of issues of interest to sociologists.

DENSITY, PATHOLOGY, AND FERTILITY: THE GALLE, GOVE, McPHERSON MODEL

As a point of departure for our discussion we selected one study for extensive review. The Galle, Gove, McPherson (GGM) study of population density and pathology (Galle et al., 1972) is both representative of empirical work on the question and a very influential study; it has been cited extensively in subsequent discussions, and has been "subjected to unusually deep scrutiny" (Choldin, 1978:101). The reanalyses of the GGM data (Ward, 1975;

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The programming for this research was done by Stephen Schmaltz, whose assistance and advice were invaluable. Daniel J. Fox assisted in the calculation of distance weights for Chicago census tracts and provided graphics files of the 1970 Chicago census tract map.

[1] For example, between 1973 and 1980 the *Social Sciences Citation Index* recorded 140 citations of Galle, Gove and McPherson (1972), one of the most influential papers in the field which we describe in detail below.

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McPherson, 1975), while raising important methodological issues, have not challenged the initial finding of a relationship between density and pathology. The essence of the GGM argument is that density has effects on pathology independent of other causes of pathology, notably social class and ethnicity. They find support for the argument in their analysis of data for Chicago's community areas. Four measures of density (number of persons per room, rooms per housing unit, housing units per structure, and structures per acre, all transformed into natural logarithms) are found to be related to five indicators of pathology (mortality ratio, fertility rate, public assistance rate, juvenile delinquency rate, and rate of admission to mental hospitals).

In this paper we focus only on the fertility relationship for two reasons. First, of the five measures of "pathology" incorporated in the GGM study, fertility is the least plausibly related to density. Choldin (1978:98) argues that it is normatively biased to assume that high fertility is pathological in the same sense that the other GGM indicators are. Furthermore, there is evidence, cited by Choldin (1978), that in other species density suppresses normal mating behavior rather than intensifying it; thus, for other species, there tends to be a negative relationship between fertility and population density. The GGM study acknowledges this inconsistency by referring to the differences between homo sapiens and other species in terms of conception possibilities, but their argument is highly speculative and is the weakest of the arguments presented for each pathology. We find the observed relationship between fertility and density most puzzling given the weak theoretical foundation on which the expectation (or the explanation) of the relationship is based.

Second, the type of model we believe is appropriate for fertility is different from those which are suitable for the other pathologies. To preview our argument, for fertility we propose a model in which spatial processes influence the distribution of the variables being studied in a way that is very different from the other pathologies. The model is described in more detail below. We turn now to a discussion of spatial autocorrelation.

SPATIAL AUTOCORRELATION

It is usually appreciated that the naive application of least-squares estimation to time-series data frequently produces "nonsense correlations" because the sampling variance of coefficients is underestimated (Yule, 1926; Granger and Newbold, 1974; Johnston, 1972). An analogous problem arises in the analysis of spatially distributed data such as census tracts, counties, and states.

One of the key assumptions in the linear regression model is that the disturbances are uncorrelated with each other.[2] That is,

$$E(U_iU_j) = 0 \text{ for all } i \neq j.$$

This means that variables which influence the dependent variable, other than the independent variables, are not systematically related to each other. When disturbances are related, they are said to be autocorrelated. In the presence of autocorrelated disturbances, ordinary least-squares performs poorly because the available observations typically provide a restricted and misleading sample of the mechanisms being studied. A simple anthropological example may help illustrate the point.[3] Raoul Naroll (1970:976) describes cultural element distribution data for California which show that the presence of patrilinear totemic clans is perfectly and positively associated with the presence of flageolets, pack frames made of sticks and cord, oval plate pottery, squared mullers, and the favoring of twins. These associations are obviously a result of the common diffusion of the traits. If one moved to another culture area where patrilinear totemic clans are found, one would not expect to find the same association. As likely as not, the association between patrilinear totemic clans and the presence of flageolets, etc., would be negative. Patrilinear totemic clans would be associated with other types of musical instruments, pack frames, pottery, mullers, and treatment of twins. The problem here is not that the correlations are biased. One is no more likely to get a positive correlation than a negative correlation. The problem is that the replications of the experiment are not independent, and when we apply the standards developed for independent replications to these data, we are easily misled. Nor is there a problem with the estimation procedure per se; any technique which assumes independence will produce estimates with relatively high variability. In the long run, if we were able to examine enough replications of the association between the traits from many parts of the world, the correlation would approach zero, since there is, in fact, no causal relationship between them. This, then, is the first consequence of autocorrelation of distur-

[2] For normally distributed variables this assumption implies independence.

[3] The anthropological literature on Galton's problem is a special case of spatial autocorrelation. For a discussion, see Blalock (1968:172), Loftin (1972) and Loftin and Ward (1981), White et al. (1981).

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bances. Least-squares estimates will be unbiased, but they will have high variability.

Another problem is that the least-squares estimates of the variability of coefficients will be biased. If the pattern of autocorrelation is positive, then the bias will be downward, and in a situation where coefficient estimates are likely to be quite poor, the formulas will indicate that the estimates are much better than they are. The problem can be illustrated with the formulas for the variance of the least-squares coefficients, which are presented in the Appendix. The use of least-squares formulas which ignore the covariance of the disturbances will underestimate the variance of the slope estimates.[4] Thus tests of significance will be biased toward rejecting the null hypothesis.

The problem of autocorrelated disturbances is well recognized and frequently discussed in the context of time series. Many texts correctly note that autocorrelation is much more common in time-series than in cross-sectional designs. However, when the units of analysis are geographic areas such as census tracts, counties, or states, autocorrelated disturbances are probably just as common in cross-sectional data as in time-series.[5] The anthropological example provided by Naroll is illustrative. The GGM model is another good example of autocorrelation where the units of analysis are geographic areas, in this case community areas of Chicago. The GGM model implicitly assumes independence of disturbances by relying on ordinary least-squares regression. This is something like assuming that events in one community area are independent of events in other community areas, so that the 75 community areas represent 75 separate, independent observations. We find this assumption unrealistic for the community areas used in this study. Indeed a fundamental principle of human ecology is that social characteristics are systematically and predictably distributed within the city. A variety of mechanisms are responsible for land use paterns in which the social characteristics of one area are directly related to those of adjacent areas. If it is true that the disturbances in the GGM model are autocorrelated, then the coefficient estimates are inefficient and the significance tests will be misleading.

Two recent papers by Doreian (1980a, 1980b) discuss spatial autocorrelation and demonstrate its effects on parameter estimates. He identifies two types of spatial models: the "spatial disturbances model," in which the effects of autocorrelation among disturbances are incorporated; and the "spatial effects model," in which the effects of autocorrelation of the dependent variable are incorporated. The choice between these two models is primarily a substantive rather than an empirical task (Doreian, 1980a:34). For our model, we incorporate spatial autocorrelation among the disturbances (Doreian's spatial disturbances model). While it might be argued that each of the other four pathologies in the GGM study is characterized by autoregression in the dependent variable, we find this least likely in the case of fertility.[6] That is, while it is probably likely that the juvenile delinquency rate in one area is causally related to that in an adjoining area (because of the activities of juvenile gangs, or similar mechanisms), there is no obvious reason that this should be the case for fertility. On the other hand, there is reason to believe that other factors associated with fertility (e.g., age structure, housing type, etc.), which would be included in the disturbance term, are related across spatial units.

DATA AND THE ORIGINAL MODEL

The data for the test of the original GGM model and the data we use here to evaluate our spatial autocorrelation model are from the *Local Community Fact Book for Chicago, 1960* (Kitagawa and Taeuber, 1963). The book reports data on a wide range of topics for each of the 75 "Community Areas" of Chicago, which are constructed on the basis of census tracts.[7]

Three types of variables are provided by the *Fact Book:* the dependent variable, fertility; the social structural variables, class and ethnicity; and the density measures.

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[4] The least-squares residuals will be smaller than the true disturbances so that the residual variance will provide a biased estimate of the variance of the disturbances. This contributes an additional bias in the same direction.

[5] When panels or pooled time-series and cross-sectional data are used it is possible, of course, to have autocorrelation in both time and space. See Granger (1969) and Pfeifer and Deutsch (1980).

[6] In fact, the other four pathologies are probably characterized by autocorrelation of both the dependent variables and the disturbances. Models in which both are autoregressive are considerably more difficult to estimate than models where only one of these operates and are beyond the scope of this paper (cf. Doreian 1980a:58). See Johnston (1972:307–20) for a discussion of estimation of this type of model with time-series data.

[7] Our results include data for all 75 areas, while the GGM test is restricted to 74 areas. Galle et al. exclude "The Loop" from all of their analyses because it is an outlier in the mental hospital admissions model. An examination of the residuals showed no such problem for fertility.

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General Fertility Rate

The number of births in a community area per 1,000 women aged 15 to 44 in the same area.

Social Class Index

A weighted combination of three indicators: (1) the percentage of employed males in the community who have white-collar occupations; (2) the median number of years of school completed by all persons 25 years of age and older in the community area; and (3) the median family income for all families residing in that area. The weights are derived from a regression of the components on each of the pathologies included in the GGM work (Galle et al., 1972:30).

Ethnicity Index

A weighted combination of three indicators: (1) the percentage of Blacks in the community area; (2) the percentage of Puerto Ricans in the community area; and (3) the percentage of foreign-born in the community area. The weights are derived in a similar fashion to the social class index weights (Galle et al., 1972:30).

Population Density

Persons per acre is decomposed into four elements: (1) the number of persons per room; (2) the number of rooms per housing unit; (3) the number of housing units per structure; and (4) the number of structures per acre. Each density is expressed in terms of its natural logarithm so that together they represent the effects of persons per acre.

Although Galle, Gove and McPherson present their results in terms of multiple partial correlations coefficients, for our purposes it is more useful to present the results in terms of the full regression model. The results (see Table 1) provide substantial evidence that the population density of the community area influences the general fertility rate. Following conventional procedures for least-squares estimates, we would conclude that each of the density variables has a statistically significant effect on fertility, and that class has a significant impact. On the other hand, ethnicity is not statistically significant when the other variables are controlled. The coefficient of determination (R^2) is .75, indicating a very good fit for the model.

Table 1. Least-Squares Estimates of the GGM Model

Variable	$\hat{B}$	$S(\hat{B})$	$\hat{B}/S(\hat{B})$
Intercept	46.04	37.96	1.21
LN Persons per Room	96.89	29.66	3.27
LN Rooms per Unit	65.88	26.47	2.49
LN Units per Structure	18.94	8.72	2.17
LN Structures per Acre	7.12	4.01	1.78
Class Index	−.065	.020	−3.22
Ethnicity Index	.003	.0038	.70
S.E.	15.08	—	—
R^2	.75	—	—

Galle, Gove, and McPherson conclude that crowding has a "serious impact on human behavior and that social scientists should consider overcrowding when attempting to explain a wide range of pathological behaviors" (1972:29). If the disturbances are autocorrelated, however, the results of the statistical analysis may be misleading and these conclusions unwarranted.

It is not possible to examine directly the disturbances since they are by definition unobserved. We can, however, investigate the distribution of the least-squares residuals which are estimates of the disturbances. By examining Figure 1, which is a map of the distribution of the residuals from the GGM model across the 75 Chicago Community Areas,[8] one can see that patches with similar levels of residual fertility fall in the same general area.[9]

The independent variables in the GGM model are also spatially autocorrelated as illustrated by Figures 2 and 3, which show the distribution of a discrete version of the logarithm of the number of rooms per housing unit and the ethnicity index.[10] The autocorrelation of the independent variables plays an important role in the variance of the estimates. It would be unusual for disturbances to be autocorrelated while independent variables are randomly distributed, but if this were the case, the problems associated with ordinary least-squares estimation would not be serious (see

[8] The map of 1960 community areas was constructed by modifying a computer file containing a map of the 1970 census tracts. We used the definitions of the community areas provided by Kitagawa and Taeuber (1963) and the specification of changes in Chicago census tracts provided by the U.S. Census (1972: Tables A and B). In a small number of cases the change in census tracts between 1960 and 1970 made it impossible to reconstruct the 1960 community exactly from the 1970 tract map. The distortions introduced, however, are trivial and can be ignored for present purposes.

[9] A test for the autocorrelation of disturbances has been developed by Cliff and Ord (1973). Additional discussion is found in Doreian (1980a) and Burridge (1980).

[10] These two variables were selected for illustrative purposes. All of the independent variables show strong patterns of autocorrelation.

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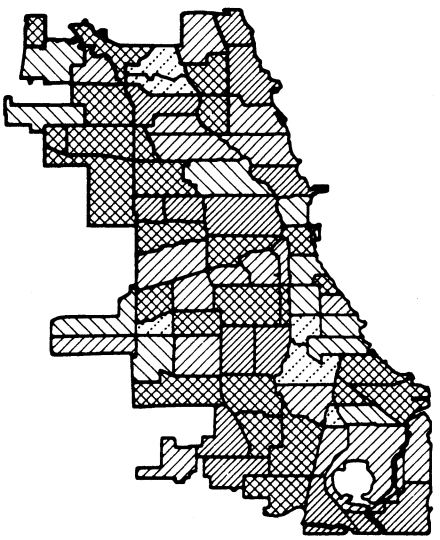

Figure 1. Map of Residuals from Least-Squares Analysis of Galle, Gove, and McPherson Model

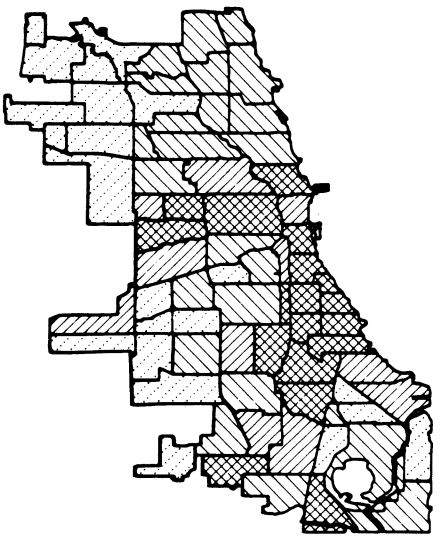

Figure 2. Map of the Logarithm of the Number of Rooms Per Housing Unit

Johnston, 1972:246, and Loftin and Ward, 1981).

SPATIAL AUTOCORRELATION MODEL

As an alternative to the GGM model, we will describe a model which assumes that the disturbances in each community area are systematically related to those in adjacent areas. In more specific terms the model is as follows:

$$\mathbf{Y} = \mathbf{XB} + \mathbf{U} \quad (1)$$
$$\mathbf{U} = \rho\mathbf{WU} + \mathbf{E} \quad (2)$$

Where **Y** is a vector of fertility rates; **X** is an $n \times (k + 1)$ matrix of predictor variables augmented by a column of ones to represent the intercept; **W** is an $n \times n$ matrix of weights which will be discussed below; **B** is a vector of coefficients; ρ is a scalar and **E** is a vector of disturbances assumed to be independently and normally distributed with mean of zero and constant variance.

The weight matrix, **W**, is used to represent the pattern of interaction between disturbances at locations i and j. This model is analogous to the first-order autoregressive model which is widely used in time-series analysis. If i were an index representing time-ordered observations and w_{ij} were set equal to one when $j = i-1$ and 0 otherwise, model (2) would be:

$$U_i = \rho U_{i-1} + E_i \quad (3)$$

which is the first-order autoregressive time-series model.

In time-series this weighting scheme is a natural choice because of the asymmetry of time order and the fact that effects generally decay over time. For spatial analysis the choice is more problematic. In this analysis we use three different sets of weights:[11]

1. Common Boundary Weights
   $w(1)_{ij} = 1$ if area j shares a common boundary with area i and 0 otherwise (for all $i \neq j$).

2. Standardized Common Boundary Weights
   $w(2)_{ij} = w(1)_{ij} / c_i$
   where $c_i = \Sigma_j w_{ij}^*$

3. Standardized Distance Weights
   $w(3)_{ij} = w_{ij}^* / c_i^*$
   where,
   $w_{ij}^* = d_{ij}^{-1}[B_{i(j)}]$,
   $c_i^* = \Sigma_j w_{ij}^*$,
   d_{ij} is the distance between the geographic centers of area i and area j,
   $B_{i(j)}$ is the proportion of the perimeter of area i which is shared with area j.

[11] For a discussion of alternative weighting schemes, see Cliff and Ord (1973). The standardized common boundary weight scheme (number two) is used by Doreian (1980a).

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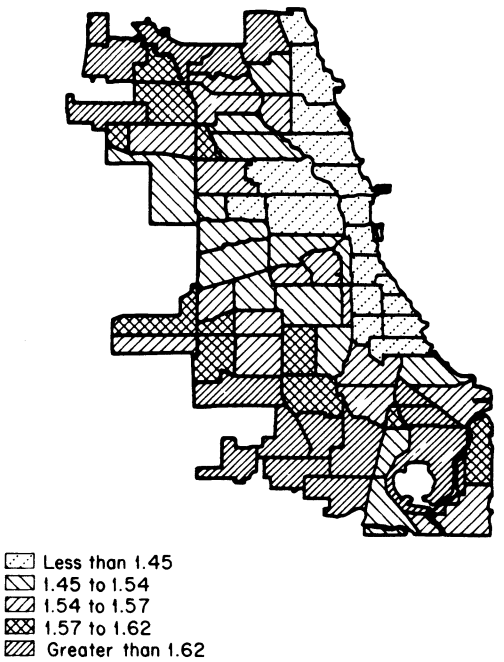

Figure 3. Map of Ethnicity Index

For all three weighting schemes, those parts of area boundaries which coincide with the boundary of the city are excluded, and when i=j we define the weight as zero.

ESTIMATION PROCEDURE

The estimation of the model is done via a modified Cochrane-Orcutt generalized difference procedure suggested by Cliff and Ord (1973) and Ord (1975).

By substituting equation (1) into (2) the model can be expressed as

$$\mathbf{Y} - \rho\mathbf{WY} = \mathbf{XB} - \rho\mathbf{WXB} + \mathbf{E} \qquad (4)$$

If $\mathbf{A} = (\mathbf{I} - \rho\mathbf{W})$, then (4) can be written

$$\mathbf{AY} = \mathbf{AXB} + \mathbf{E} \qquad (5)$$

Since **E** is independent and identically distributed with zero mean and constant variance, least-squares estimates of **B** will be best linear unbiased. We cannot, however, use this approach directly since we do not know the value of ρ. Instead we adapt the iterative procedure outlined by Cliff and Ord, which is analogous to the Cochrane and Orcutt (1949) procedure:

1. Least squares residuals ($\hat{\mathbf{U}}$ are calculated using (1).
2. ρ is estimated from $\hat{\mathbf{U}} = \rho\mathbf{W}\hat{\mathbf{U}} + \mathbf{E}$ using a maximum likelihood procedure outlined by Ord (1975).
3. $\mathbf{A} = \mathbf{I} - \rho\mathbf{W}$ is used to transform **X** and **Y**.
4. The transformed matrices, $\hat{\mathbf{X}}$ and $\hat{\mathbf{Y}}$, are used to derive least-squares estimates of **B**.
5. New residuals are calculated using the new estimate of **B** and the process begins again.

The procedure is repeated until estimates converge. An asymptotic variance-covariance matrix for the model is calculated following Ord (1975:125).

RESULTS

The parameter estimates for the autoregressive disturbance model (see Table 2) are different from those based on the GGM model. With all three of the weight matrices, the coefficients of the population density variables are substantially reduced. The standardized distance weights probably provide the best estimates since they take both distance and the relative size of common boundary into account, but the major results are similar regardless of which weight specification is used. The coefficient estimates for the density variables are much smaller in the spatial autocorrelation models than in the GGM model. Using the standardized distance weights for comparison, the density coefficient estimates are reduced in the autocorrelation model (30 percent for persons per room, 34 percent for rooms per housing unit, 31 percent for housing units per structure, and 17 percent for structures per acre). In contrast the coefficient estimates for the class index remain about the same, while they increase for the ethnicity index.

If we test each of the coefficient estimates in the standardized distance specification against the null hypothesis, B = 0, we would conclude that the only statistically significant effect of density on fertility is for persons per room, but that both class and ethnicity are highly significant.

The results for the standardized common boundary weights are essentially equivalent to those for the standardized distance weights, while the results for the remaining weight specification (common boundary) are slightly different. Structures per acre is the one significant effect for the density variables using common boundary weights.[12] Because the standardized distance weights use more information and because an examination of maps of the residuals from this model indicates the best approximation of a random spatial series, we conclude

[12] We conducted one-tailed tests (H_0: $B \leq 0$). Since Galle et al. did not predict the sign, a two-tailed test would have been reasonable. Nevertheless, we conducted one-tailed tests, giving the GGM model the benefit of the doubt.

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Table 2. Estimates for Spatial Autocorrelated Fertility Model

Variable	Common Boundary Weights			Standardized Common Boundary Weights			Standardized Distance Weights		
	$\hat{B}$	$S(\hat{B})$	$\hat{B}/S(\hat{B})$	$\hat{B}$	$S(\hat{B})$	$\hat{B}/S(\hat{B})$	$\hat{B}$	$S(\hat{B})$	$\hat{B}/S(\hat{B})$
Intercept	158.26	46.51	3.40	71.58	39.62	1.81	70.82	38.97	1.82
LN Persons per Room	17.73	30.58	.58	64.01	29.38	2.18	67.75	29.26	2.32
LN Rooms per Unit	−18.41	32.18	−.57	42.10	27.36	1.54	43.34	27.21	1.59
LN Units per Structure	−1.87	9.41	−.20	12.61	9.45	1.33	13.04	9.17	1.42
LN Structures per Acre	7.19	4.04	1.78	6.10	4.12	1.48	5.91	4.13	1.45
Class Index	−.0752	.0222	−3.38	−.0603	.0213	−2.83	−.0606	.0208	−2.91
Ethnicity Index	.0103	.0030	3.43	.0083	.0039	2.13	.0082	.0039	2.09
RHO	.61	—	—	.35	—	—	.30	—	—
S.E.	19.74	—	—	13.90	—	—	13.98	—	—

that these are the best estimates of the model. Therefore we argue that the data fit a model in which there are meaningful effects of ethnicity and class on fertility, but that the correlation between density and fertility, except for persons per room, is spurious, a result of spatially autocorrelated disturbances.

DISCUSSION AND CONCLUSIONS

The contrast between the analysis based on the GGM model and the spatial autocorrelation model indicates that the ordinary least-squares estimates are misleading. Our interpretation is that the true value for three of the density coefficients is zero. Therefore the more efficient estimates based on the autoregressive model are smaller, that is, closer to the true value. In contrast, the coefficient estimates for ethnicity and class remain unchanged or increase. This is as expected since increased efficiency would not necessarily lead one to expect smaller coefficients where the true values are not zero.

The less marked changes in the estimates of standard errors are consistent with the increased efficiency. Once the efficiency of the coefficient estimates is improved, we would not expect equally compensating changes in the standard error estimates.[13] The crucial comparison to make is between the inferences one would draw from each set of results; our findings show that the inferences differ depending on the estimation procedure. The GGM model, which includes no information on spatial interaction, produces results which differ from our three models, each of which incorporates some type of information on spatial distribution of the data.

The analysis leads us to three conclusions. First, the GGM findings with regard to fertility are an artifact of the failure to recognize the presence of disturbance variables which are spatially autocorrelated. When the model includes spatially autocorrelated disturbances, there is only one statistically significant effect of density on fertility.

Second, the fact that the estimates of the effects of population density on fertility change when the effects of spatial processes are included in the model raises serious question about the validity of the other results reported by GGM and other studies of density based on geographically defined units. We can not address these other findings directly since we have not yet developed alternative estimates. Nevertheless, our estimates for fertility provide reason to suspect that the inclusion of spatial processes would change all of the GGM estimates substantially.

Finally, our research illustrates the importance of spatial mechanisms in modeling social processes. The GGM analysis is only one of many examples of studies which use geographically defined areas without due consideration to interactions between units. The contrast between the GGM estimates and our spatial autocorrelation estimates suggests that a wide range of research findings should be reexamined to consider the effects of spatial processes.

[13] The large improvement in the efficiency of the slope estimates and the relatively small change in the estimates of standard errors is consistent with small-sample simulations (see Loftin and Ward, 1981; Rao and Griliches, 1969).

APPENDIX

The usual formula for the variance of the coefficients is:

$$\text{Var}(\mathbf{B}) = (\mathbf{X}'\mathbf{X})^{-1}\,\mathbf{X}'\mathbf{E}(\mathbf{U}\mathbf{U}')\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1},$$

where $\mathbf{X}$ is a matrix of independent variables and $\mathbf{U}$ is a vector of disturbances.

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When the disturbances are not autocorrelated and have constant variance, the variance-covariance matrix of the disturbances has the simple form of a scalar times an identity matrix:

$$E(\mathbf{UU}') = \sigma^2\mathbf{I}.$$

Therefore, the variance of the coefficients is simply:

$$\text{Var}(\mathbf{B}) = \sigma^2 (\mathbf{X}'\mathbf{X})^{-1}.$$

Or for a bivariate regression:

$$\text{Var}(B) = \sigma^2/\Sigma_i(X_i - \bar{X})^2$$

But if the disturbances are autocorrelated, the off-diagonal elements in the disturbance variance-covariance matrix will not be zero, and the variance of the coefficients does not reduce to this simple form. Instead, it is a more complex expression which reflects the covariance between the disturbances:

$$\text{Var}(\mathbf{B}) = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{V}\mathbf{X}\,(\mathbf{X}'\mathbf{X})^{-1}$$

where **V** is the variance-covariance matrix of the disturbances.

If **X** and **U** are both positively autocorrelated, which is a typical pattern, then it can be shown that

$$\text{Var}(B) > \sigma^2 (\mathbf{X}'\mathbf{X})^{-1}.$$

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